

# Effective meson-meson interaction in 2+1 dimensional lattice QED \*

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A definition of an effective meson-meson interaction adapted to the framework of a lattice simulation is presented. Results, based on a truncated momentum-space 4-point time correlation matrix, and preliminary data from a complementary coordinate-space simulation, are shown.

## 1. INTRODUCTION

The nucleon-nucleon interaction has been parametrized by boson-exchange models for several decades. By their very ansatz, boson-exchange models lack the possibility of giving insight into the nature of the nucleon-nucleon interaction and its underlying physics from first principles.

From the viewpoint of lattice QCD a study of effective hadron-hadron interaction opens up mainly uncharted territory. Only a few attempts have been made to break new ground [1,2] in a field which could be characterized as Lattice Nuclear Physics. This field would concern itself mostly with the *effective forces between composite colour singlets*, in other words, their "chemistry". This goes along with a characteristic *change of scale* leaving a residual effective interaction  $10^{-2}$  to  $10^{-3}$  times smaller than a typical hadron mass.

The numerical difficulties involved in going from the GeV to the MeV scale on the lattice are evident. For this reason it seems appropriate to investigate simpler lattice models which exhibit similar physics. Most importantly, of course, one needs to insist on confinement and the presence of fermions.

It is in this spirit that we report on a simulation aimed at extracting an effective meson-meson interaction from a QED gauge field model in 2+1 dimensions with staggered fermions which has confinement. In an earlier related simulation

[3] scattering phase shifts were obtained for this system based on Lüscher's proposal. Regarding the parameters of the QED<sub>2+1</sub> model and numerical strategies we refer the reader to Ref. [3].

## 2. MESON-MESON CORRELATOR

An appropriate set of operators can be constructed from one-meson fields with momentum  $\vec{p} = \frac{2\pi}{L}(k_1, k_2)$

$$\phi_{\vec{p}}(t) = L^{-2} \sum_{\vec{x}} e^{i\vec{p} \cdot \vec{x}} \bar{\chi}_d(\vec{x}t) \chi_u(\vec{x}t) \quad (1)$$

where  $\chi$  and  $\bar{\chi}$  are staggered Grassmann fields with external flavours  $u$  and  $d$ . Meson-meson fields with total momentum  $\vec{P} = 0$  then are

$$\Phi_{\vec{P}}(t) = \phi_{-\vec{P}}(t) \phi_{+\vec{P}}(t). \quad (2)$$

The information about the dynamics of the meson-meson system is contained in the 4-point time correlation matrix

$$C_{\vec{p}\vec{q}}^{(4)}(t, t_0) = \langle \Phi_{\vec{p}}^\dagger(t) \Phi_{\vec{q}}(t_0) \rangle - \langle \Phi_{\vec{p}}^\dagger(t) \rangle \langle \Phi_{\vec{q}}(t_0) \rangle. \quad (3)$$

Working out the contractions between the Grassmann fields the following diagrammatic representation is obtained

$$C^{(4)} = C^{(4A)} + C^{(4B)} - C^{(4C)} - C^{(4D)} \quad (4)$$

$$= \parallel \parallel + \times - \bowtie - \bowtie. \quad (5)$$

The quark-exchange diagrams  $C^{(4C)}$  and  $C^{(4D)}$  are a source of interaction between the mesons, but then, so are diagrams  $C^{(4A)}$  and  $C^{(4B)}$ . Denoting contractions of the Grassmann fields by

$$\overset{n}{\chi} \dots \overset{n}{\chi} = G_n \quad \text{with} \quad n = 1 \dots 4 \quad (6)$$

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we have for example

$$C^{(4A)} = \langle \phi_{\vec{q}}^{\dagger} \phi_{-\vec{q}}^{\dagger} \phi_{-\vec{p}} \phi_{\vec{p}} \rangle \sim \langle G_1 G_2^* G_3 G_4^* \rangle \quad (7)$$

where  $\sim$  stands for four Fourier sums that carry over from (1). Note that the gauge configuration average is taken over the product of all four propagators  $G$ . This is a source of gluonic interaction.

In order to define an effective meson-meson interaction it is crucial that the noninteracting components in  $C^{(4)}$  be isolated. This is achieved by means of a cumulant (or cluster) expansion [4] of the gauge field average. For example

$$C^{(4A)} \sim \langle G_1 G_2^* \rangle \langle G_3 G_4^* \rangle + \langle G_1 G_3 \rangle \langle G_2^* G_4^* \rangle + \langle G_1 G_4^* \rangle \langle G_3 G_2^* \rangle + \langle G_1 G_2^* G_3 G_4^* \rangle_c \quad (8)$$

where the last term defines the cumulant. The first term is readily interpreted to mean free, uncorrelated, mesons on the lattice. With reference to (6)–(8) define

$$\bar{C}^{(4A)} = \langle \phi_{-\vec{q}}^{\dagger} \phi_{-\vec{p}} \rangle \langle \phi_{\vec{q}}^{\dagger} \phi_{\vec{p}} \rangle \sim \langle G_1 G_2^* \rangle \langle G_3 G_4^* \rangle. \quad (9)$$

Diagram  $C^{(4B)}$  can be analyzed in a similar fashion, which then leads us to define  $\bar{C}^{(4B)}$ . The sum of those

$$\bar{C}^{(4)} = \bar{C}^{(4A)} + \bar{C}^{(4B)} \quad (10)$$

constitutes the free meson-meson correlator with the *relative* interaction switched off.

### 3. EFFECTIVE INTERACTION

The deviation of  $C^{(4)}$  from  $\bar{C}^{(4)}$  somehow contains the residual effective meson-meson interaction. To find its definition, one may calculate the correlation matrix  $\hat{C}$  for an elementary boson field  $\hat{\phi}(\vec{x}t)$  that lives on the lattice sites and is subject to a perturbation,  $\hat{H} = \hat{H}_0 + \hat{H}_I$ . It is possible to derive an explicit expression for  $\hat{H}_I$  in terms of the correlators of order  $N = 0$  and  $N = 1$  from the perturbative expansion of  $\hat{C}$ . This result leads us, by way of analogy, to define an effective meson-meson interaction as

$$\mathcal{H}_I = -\frac{\partial}{\partial t} \left[ \bar{C}^{(4)-\frac{1}{2}} C^{(4)} \bar{C}^{(4)-\frac{1}{2}} \right]_{t=t_0}. \quad (11)$$

This definition is consistent with  $\mathcal{C} = e^{-\mathcal{H}_I(t-t_0)}$  where  $\mathcal{C}$  is the matrix inside [...] in (11).

### 4. RESULTS

The simulation has been done on an  $L^2 \times T = 24^2 \times 32$  lattice with the compact  $U(1)$  Wilson action at  $\beta = 1.5$  and quenched staggered fermions. The correlation matrices  $C^{(4)}$  and  $\bar{C}^{(4)}$  were computed for a truncated set of momenta  $\vec{p} = \frac{2\pi}{L}(k_1, k_2)$  with  $|k_{1,2}| \leq 2$ , and for time slices  $t = 15 \dots 19$  around the symmetry point  $t_c = 17$ . Invariance of the correlators  $C^{(4)}$  and  $\bar{C}^{(4)}$  under the lattice symmetry group  $O(2, Z)$  was utilized. Its irreducible representations  $\Gamma = A_1, A_2, B_1, B_2$  (see [3]) render the reduced matrix  $\bar{C}^{(4;\Gamma)}$  diagonal, and positive, which makes computing the inverse square root  $\bar{C}^{(4;\Gamma)-\frac{1}{2}}$  trivial. The time derivative in (11) was computed from cosh-fits to the eigenvalues of  $\mathcal{C}_{p,q}^{(4;\Gamma)}(t, t_0)$ . Finally, a Fourier transformation, back to coordinate space, and a partial-wave projection were performed

$$\langle \vec{r} | \mathcal{H}_I | \vec{r}' \rangle = \sum_{t=0}^{\infty} \mathcal{H}_I^{(t)}(r, r') P_t(\varphi - \varphi') + \dots \quad (12)$$

where  $P_t(\varphi) \propto \cos(\ell\varphi)$  and  $r, \varphi$  are the polar coordinates of  $\vec{r}$ . The  $s$ -wave effective interaction ( $\ell = 0$ ) is the dominant contribution to (12). Its elements  $\mathcal{H}_I^{(0)}(r, r')$  are displayed in Fig. 1. Repulsion at small relative separation and attraction at larger distance are evident. This is consistent with the results of [3]. It should be noted that the small momenta considered in the correlation matrices cannot resolve short distances well. In particular, the repulsive core inevitably comes out soft and broad.

### 5. COORDINATE-SPACE HARD CORE

A complementary simulation which starts directly from coordinate-space matrix elements of the correlators  $\bar{C}^{(4)}$  and  $C^{(4)}$  reveals an interesting perspective on the repulsive core. Those correlator matrix elements are constructed from meson-meson fields with relative distance  $\vec{r}$

$$\Phi_{\vec{r}}(t) = \sum_{\vec{p}} e^{-i\vec{p} \cdot \vec{r}} \phi_{-\vec{p}}(t) \phi_{\vec{p}}(t). \quad (13)$$

From the resulting expression for  $C^{(4)}$  in terms of the fermion propagator it is easy to see *analytically* that  $C_{\vec{r}\vec{r}'}^{(4)}(t, t_0) = 0$  if  $\vec{r} = 0$  or  $\vec{r}' = 0$ . To

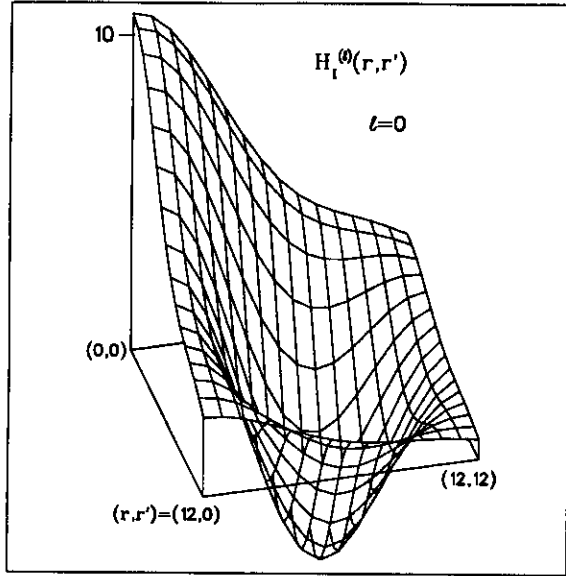


Figure 1. Nonlocal  $s$ -wave effective interaction between two mesons. The vertical scale is in units of  $10^{-3}a^{-3}$ .

the extent that the energy  $W(\vec{r})$  of the meson-meson system at fixed relative distance  $\vec{r}$  can be extracted from the large- $t$  behavior of the diagonal elements

$$C_{\vec{r}\vec{r}}^{(4)}(t, t_0) \simeq c(\vec{r})e^{-W(\vec{r})(t-t_0)} \quad (14)$$

we then may conclude that  $W(\vec{r} = 0) = +\infty$ , provided the strength factor  $c(\vec{r})$  is non-zero at  $\vec{r} = 0$ . Thus the effective interaction possesses a hard repulsive core. It is due to the anticommuting nature of the fermion field (Pauli repulsion). In Fig. 2 we show raw simulation data for  $W(\vec{r})$ . A careful interpretation of those results will be forthcoming [5]. It should be emphasized that Fig. 1 and Fig. 2 are based on incompatible approximation schemes and cannot be compared in quantitative terms.

## 6. CONCLUSION

A practical method to extract an effective hadron-hadron interaction has been developed

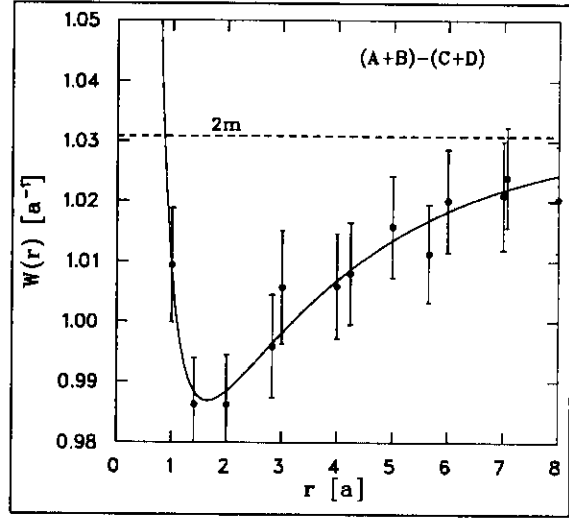


Figure 2. Fixed-distance meson-meson energies including all four diagrams of (5). The dashed line indicates the mass of two free mesons.

and applied within the framework of a  $QED_{2+1}$  lattice model. (It replaces an earlier proposal [6] which has proved numerically unsuitable.) Its momentum-space version yields a fully dynamical effective interaction with short-range repulsion and intermediate-range attraction, the hallmark features of the nucleon-nucleon interaction. A complementary coordinate-space approach indicates a hard repulsive core for this model.

## REFERENCES

1. Y. Kuramashi, M. Fukugita, H. Mino, M. Okawa and A. Ukawa, Nucl. Phys. B (Proc. Suppl.) 34 (1994) 117.
2. K. Rabitsch, H. Markum and W. Sakuler, Phys. Lett. B318 (1993) 507.
3. H.R. Fiebig, R.M. Woloshyn and A. Dominguez, Nucl. Phys. B418 (1994) 649.
4. S.K. Ma, Statistical Mechanics, World Scientific, Philadelphia - Singapore, 1985.
5. H.R. Fiebig and H. Markum, in preparation.
6. J.D. Canosa and H.R. Fiebig, Nucl. Phys. B (Proc. Suppl.) 34 (1994) 561.